

# Factorising expressions

## A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

## Key points

- Factorising an expression is the opposite of expanding the brackets.
- A quadratic expression is in the form  $ax^2 + bx + c$ , where  $a \neq 0$ .
- To factorise a quadratic equation find two numbers whose sum is  $b$  and whose product is  $ac$ .
- An expression in the form  $x^2 - y^2$  is called the difference of two squares. It factorises to  $(x - y)(x + y)$ .

## Examples

**Example 1** Factorise  $15x^2y^3 + 9x^4y$

$$15x^2y^3 + 9x^4y = 3x^2y(5y^2 + 3x^2)$$

The highest common factor is  $3x^2y$ .  
So take  $3x^2y$  outside the brackets and then divide each term by  $3x^2y$  to find the terms in the brackets

**Example 2** Factorise  $4x^2 - 25y^2$

$$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$$

This is the difference of two squares as the two terms can be written as  $(2x)^2$  and  $(5y)^2$

**Example 3** Factorise  $x^2 + 3x - 10$

$$b = 3, ac = -10$$

$$\text{So } x^2 + 3x - 10 = x^2 + 5x - 2x - 10$$

$$= x(x + 5) - 2(x + 5)$$

$$= (x + 5)(x - 2)$$

- 1 Work out the two factors of  $ac = -10$  which add to give  $b = 3$  (5 and -2)
- 2 Rewrite the  $b$  term ( $3x$ ) using these two factors
- 3 Factorise the first two terms and the last two terms
- 4  $(x + 5)$  is a factor of both terms

**Example 4** Factorise  $6x^2 - 11x - 10$

<p><math>b = -11, ac = -60</math></p> <p>So</p> $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$ $= 3x(2x - 5) + 2(2x - 5)$ $= (2x - 5)(3x + 2)$	<ol style="list-style-type: none"> <li><b>1</b> Work out the two factors of <math>ac = -60</math> which add to give <math>b = -11</math> (<math>-15</math> and <math>4</math>)</li> <li><b>2</b> Rewrite the <math>b</math> term (<math>-11x</math>) using these two factors</li> <li><b>3</b> Factorise the first two terms and the last two terms</li> <li><b>4</b> <math>(2x - 5)</math> is a factor of both terms</li> </ol>
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**Example 5** Simplify  $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$

$\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$ <p>For the numerator: <math>b = -4, ac = -21</math></p> <p>So</p> $x^2 - 4x - 21 = x^2 - 7x + 3x - 21$ $= x(x - 7) + 3(x - 7)$ $= (x - 7)(x + 3)$ <p>For the denominator: <math>b = 9, ac = 18</math></p> <p>So</p> $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$ $= 2x(x + 3) + 3(x + 3)$ $= (x + 3)(2x + 3)$ <p>So</p> $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9} = \frac{(x - 7)(x + 3)}{(x + 3)(2x + 3)}$ $= \frac{x - 7}{2x + 3}$	<ol style="list-style-type: none"> <li><b>1</b> Factorise the numerator and the denominator</li> <li><b>2</b> Work out the two factors of <math>ac = -21</math> which add to give <math>b = -4</math> (<math>-7</math> and <math>3</math>)</li> <li><b>3</b> Rewrite the <math>b</math> term (<math>-4x</math>) using these two factors</li> <li><b>4</b> Factorise the first two terms and the last two terms</li> <li><b>5</b> <math>(x - 7)</math> is a factor of both terms</li> <li><b>6</b> Work out the two factors of <math>ac = 18</math> which add to give <math>b = 9</math> (<math>6</math> and <math>3</math>)</li> <li><b>7</b> Rewrite the <math>b</math> term (<math>9x</math>) using these two factors</li> <li><b>8</b> Factorise the first two terms and the last two terms</li> <li><b>9</b> <math>(x + 3)</math> is a factor of both terms</li> <li><b>10</b> <math>(x + 3)</math> is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1</li> </ol>
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## Practice

1 Factorise.

a  $6x^4y^3 - 10x^3y^4$

c  $25x^2y^2 - 10x^3y^2 + 15x^2y^3$

b  $21a^3b^5 + 35a^5b^2$

2 Factorise

a  $x^2 + 7x + 12$

c  $x^2 - 11x + 30$

e  $x^2 - 7x - 18$

g  $x^2 - 3x - 40$

b  $x^2 + 5x - 14$

d  $x^2 - 5x - 24$

f  $x^2 + x - 20$

h  $x^2 + 3x - 28$

3 Factorise

a  $36x^2 - 49y^2$

c  $18a^2 - 200b^2c^2$

b  $4x^2 - 81y^2$

4 Factorise

a  $2x^2 + x - 3$

c  $2x^2 + 7x + 3$

e  $10x^2 + 21x + 9$

b  $6x^2 + 17x + 5$

d  $9x^2 - 15x + 4$

f  $12x^2 - 38x + 20$

5 Simplify the algebraic fractions.

a  $\frac{2x^2 + 4x}{x^2 - x}$

c  $\frac{x^2 - 2x - 8}{x^2 - 4x}$

e  $\frac{x^2 - x - 12}{x^2 - 4x}$

b  $\frac{x^2 + 3x}{x^2 + 2x - 3}$

d  $\frac{x^2 - 5x}{x^2 - 25}$

f  $\frac{2x^2 + 14x}{2x^2 + 4x - 70}$

6 Simplify

a  $\frac{9x^2 - 16}{3x^2 + 17x - 28}$

c  $\frac{4 - 25x^2}{10x^2 - 11x - 6}$

b  $\frac{2x^2 - 7x - 15}{3x^2 - 17x + 10}$

d  $\frac{6x^2 - x - 1}{2x^2 + 7x - 4}$

### Hint

Take the highest common factor outside the bracket.

## Extend

7 Simplify  $\sqrt{x^2 + 10x + 25}$

8 Simplify  $\frac{(x+2)^2 + 3(x+2)^2}{x^2 - 4}$

**Answers**

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|----------|------------------------------------|---------------------------------|
| <b>1</b> | <b>a</b> $2x^3y^3(3x - 5y)$        | <b>b</b> $7a^3b^2(3b^3 + 5a^2)$ |
|          | <b>c</b> $5x^2y^2(5 - 2x + 3y)$    |                                 |
| <b>2</b> | <b>a</b> $(x + 3)(x + 4)$          | <b>b</b> $(x + 7)(x - 2)$       |
|          | <b>c</b> $(x - 5)(x - 6)$          | <b>d</b> $(x - 8)(x + 3)$       |
|          | <b>e</b> $(x - 9)(x + 2)$          | <b>f</b> $(x + 5)(x - 4)$       |
|          | <b>g</b> $(x - 8)(x + 5)$          | <b>h</b> $(x + 7)(x - 4)$       |
| <b>3</b> | <b>a</b> $(6x - 7y)(6x + 7y)$      | <b>b</b> $(2x - 9y)(2x + 9y)$   |
|          | <b>c</b> $2(3a - 10bc)(3a + 10bc)$ |                                 |
| <b>4</b> | <b>a</b> $(x - 1)(2x + 3)$         | <b>b</b> $(3x + 1)(2x + 5)$     |
|          | <b>c</b> $(2x + 1)(x + 3)$         | <b>d</b> $(3x - 1)(3x - 4)$     |
|          | <b>e</b> $(5x + 3)(2x + 3)$        | <b>f</b> $2(3x - 2)(2x - 5)$    |
| <b>5</b> | <b>a</b> $\frac{2(x+2)}{x-1}$      | <b>b</b> $\frac{x}{x-1}$        |
|          | <b>c</b> $\frac{x+2}{x}$           | <b>d</b> $\frac{x}{x+5}$        |
|          | <b>e</b> $\frac{x+3}{x}$           | <b>f</b> $\frac{x}{x-5}$        |
| <b>6</b> | <b>a</b> $\frac{3x+4}{x+7}$        | <b>b</b> $\frac{2x+3}{3x-2}$    |
|          | <b>c</b> $\frac{2-5x}{2x-3}$       | <b>d</b> $\frac{3x+1}{x+4}$     |
| <b>7</b> | $(x + 5)$                          |                                 |
| <b>8</b> | $\frac{4(x+2)}{x-2}$               |                                 |